# Comp-3150: Database Management Systems

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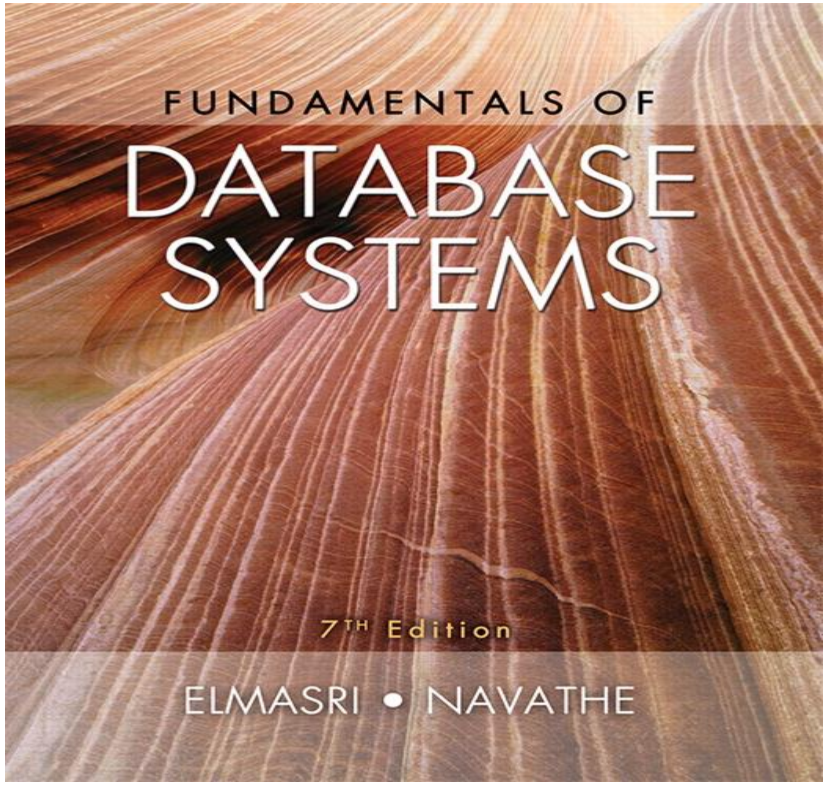
## Chapter 8:
## The Relational Algebra and
## The Relational Calculus



# Chapter 8: The Relational Algebra and The Relational Calculus: Outline

- 1. Relational Algebra
  - 1.1 Unary Relational Operations
    - (SELECT (symbol: $\sigma$ (sigma))
    - PROJECT (symbol: $\pi$ (pi))
    - RENAME (symbol: $\rho$ (rho))
  - 1.2 Binary Relational Operations
    - JOIN (several variations of JOIN exist)( $\bowtie_{\text{<joincondition>}}$ )
    - DIVISION ( $\div$ )
  - 1.3 Relational Algebra Operations From Set Theory
    - UNION ( $\cup$ ), INTERSECTION ( $\cap$ ), DIFFERENCE (or MINUS, **–** )
    - CARTESIAN PRODUCT ( **x** )
  - 1.4 Additional Relational Operations (not fully discussed)
  - 1.5 Examples of Queries in Relational Algebra
- 2. Relational Calculus
  - 2.1 Tuple Relational Calculus

# 1. Relational Algebra

- The formal languages for the relational model are:
  - the relational algebra and relational calculus.
- A data model must have a set of operations for manipulating its data structure and constraints.
- The basic set of operations for the relational model is:
  - the relational algebra which expresses the data retrieval requests as relational algebra expressions.
- A sequence of relational algebra operations is a relational algebra expression,
  - which produces a relation result that is result of a database query.

# 1. Relational Algebra

- Thus relational algebra provides:
  - (1) a formal foundation for relational model operations.
  - (2) It is used for query processing and optimization.
  - (3) Some of its concepts are implemented in the RDBMSs.
- The relational <u>calculus</u> provides a declarative (rather than procedural) language for specifying relational queries
  - as it tells what the query result should be and not how or sequence of steps for retrieving it.
- The relational algebra has two groups of operations

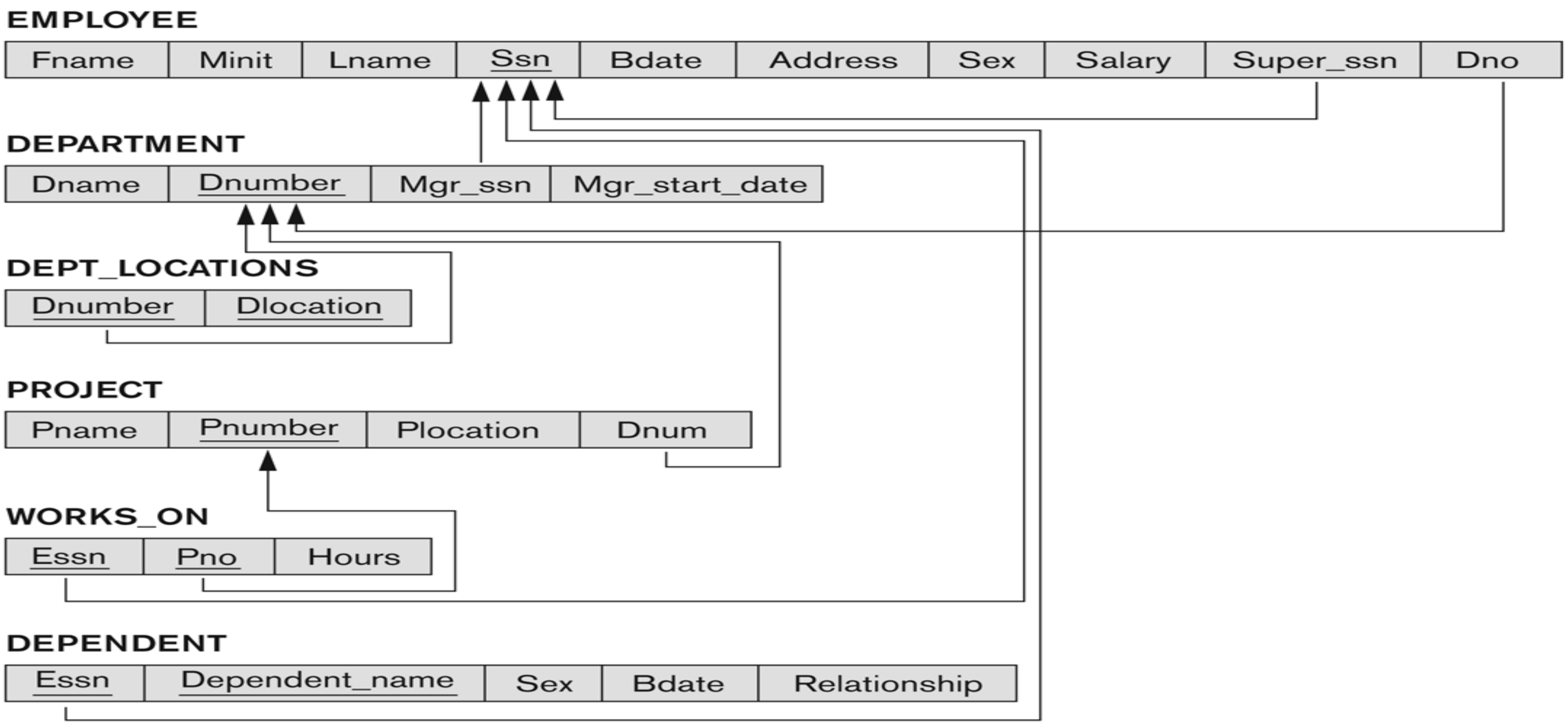
# 1. Relational Algebra

- 1.1. Unary Relational Operations
  - SELECT (symbol: $\sigma$ (sigma))
  - PROJECT (symbol: $\pi$ (pi))
  - RENAME (symbol: $\rho$ (rho))
- 1.2. Binary Relational Operations
  - JOIN (several variations of JOIN exist) ( $\bowtie_{\text{<joincondition>}}$)
  - DIVISION ($\div$ )
- 1.3. Relational Algebra Operations From Set Theory
  - UNION ( $\cup$ ), INTERSECTION ( $\cap$ ), DIFFERENCE (or MINUS, – )
  - CARTESIAN PRODUCT ( **x** )
- 1.4. Additional Relational Operations
  - OUTER JOINS, OUTER UNION
  - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

# Database State for COMPANY

- All examples discussed below refer to the COMPANY database shown here.

**Figure 5.7**
Referential integrity constraints displayed on the COMPANY relational database schema.



# 1.1 Unary Relational Operations: SELECT

- The SELECT operation (denoted by $\sigma$ (sigma)) is used to select a *subset* of the tuples from a relation based on a **selection condition**.
  - The selection condition acts as a **filter**
  - Keeps only those tuples that satisfy the qualifying condition
  - Tuples satisfying the condition are *selected* whereas the other tuples are discarded (*filtered out*)
- Examples:
  - Select the EMPLOYEE tuples whose department number is 4:

$$\sigma_{\text{DNO} = 4}\ (\text{EMPLOYEE})$$

  - Select the employee tuples whose salary is greater than $30,000:

$$\sigma_{\text{SALARY} > 30{,}000}\ (\text{EMPLOYEE})$$

# 1.1 Unary Relational Operations: SELECT

- In general, the *select* operation is denoted by $\sigma_{\text{<selection condition>}}(R)$ where
  - the symbol **σ** (sigma) is used to denote the *select* operator
  - the selection condition is a Boolean (conditional) expression specified on the attributes of relation R
  - tuples that make the condition **true** are selected
    - appear in the result of the operation
  - tuples that make the condition **false** are filtered out
    - discarded from the result of the operation

# 1.1 Unary Relational Operations: SELECT

- SELECT Operation Properties
  - The SELECT operation $\sigma_{\text{<selection condition>}}(R)$ produces a relation S that has the same schema (same attributes) as R
  - SELECT $\sigma$ is commutative:
    - $\sigma_{\text{<condition1>}}(\sigma_{\text{< condition2>}}(R)) = \sigma_{\text{<condition2>}}(\sigma_{\text{< condition1>}}(R))$
  - Because of commutativity property, a cascade (sequence) of SELECT operations may be applied in any order:
    - $\sigma_{\text{<cond1>}}(\sigma_{\text{<cond2>}}(\sigma_{\text{<cond3>}}(R)) = \sigma_{\text{<cond2>}}(\sigma_{\text{<cond3>}}(\sigma_{\text{<cond1>}}(R)))$
  - A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:
    - $\sigma_{\text{<cond1>}}(\sigma_{\text{< cond2>}}(\sigma_{\text{<cond3>}}(R)) = \sigma_{\text{<cond1> AND < cond2> AND < cond3>}}(R)))$
  - The number of tuples in the result of a SELECT is less than (or equal to) the number of tuples in the input relation R

# The following query results refer to this database state

**Figure 5.6**
One possible database state for the COMPANY relational database schema.

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|---|---|---|---|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

**DEPT_LOCATIONS**

| Dnumber | Dlocation |
|---|---|
| 1 | Houston |
| 4 | Stafford |
| 5 | Bellaire |
| 5 | Sugarland |
| 5 | Houston |

**WORKS_ON**

| Essn | Pno | Hours |
|---|---|---|
| 123456789 | 1 | 32.5 |
| 123456789 | 2 | 7.5 |
| 666884444 | 3 | 40.0 |
| 453453453 | 1 | 20.0 |
| 453453453 | 2 | 20.0 |
| 333445555 | 2 | 10.0 |
| 333445555 | 3 | 10.0 |
| 333445555 | 10 | 10.0 |
| 333445555 | 20 | 10.0 |
| 999887777 | 30 | 30.0 |
| 999887777 | 10 | 10.0 |
| 987987987 | 10 | 35.0 |
| 987987987 | 30 | 5.0 |
| 987654321 | 30 | 20.0 |
| 987654321 | 20 | 15.0 |
| 888665555 | 20 | NULL |

**PROJECT**

| Pname | Pnumber | Plocation | Dnum |
|---|---|---|---|
| ProductX | 1 | Bellaire | 5 |
| ProductY | 2 | Sugarland | 5 |
| ProductZ | 3 | Houston | 5 |
| Computerization | 10 | Stafford | 4 |
| Reorganization | 20 | Houston | 1 |
| Newbenefits | 30 | Stafford | 4 |

**DEPENDENT**

| Essn | Dependent_name | Sex | Bdate | Relationship |
|---|---|---|---|---|
| 333445555 | Alice | F | 1986-04-05 | Daughter |
| 333445555 | Theodore | M | 1983-10-25 | Son |
| 333445555 | Joy | F | 1958-05-03 | Spouse |
| 987654321 | Abner | M | 1942-02-28 | Spouse |
| 123456789 | Michael | M | 1988-01-04 | Son |
| 123456789 | Alice | F | 1988-12-30 | Daughter |
| 123456789 | Elizabeth | F | 1967-05-05 | Spouse |

# Unary Relational Operations: PROJECT

- PROJECT Operation is denoted by $\pi$ (pi)
- This operation keeps certain *columns* (attributes) from a relation and discards the other columns.
- PROJECT creates a vertical partitioning
  - The list of specified columns (attributes) is kept in each tuple
  - The other attributes in each tuple are discarded
- Example: To list each employee's first and last name and salary, the following is used:
- $\pi_{\text{LNAME, FNAME,SALARY}}(\text{EMPLOYEE})$
- This can be renamed as:
  R(Last_name, First_name, Salary) $\leftarrow$

$$\pi_{\text{LNAME, FNAME,SALARY}}(\text{EMPLOYEE})$$

# Unary Relational Operations: PROJECT (also Rename)

- We can define a formal RENAME operation ($\rho$ ) to rename either the relation or attribute names or both.
- The general RENAME operation applied to a relation $R(A_1, A_2, \ldots, A_n)$ of degree n is of the form:
- (i) $\rho_{S(B1, B2, \ldots, Bn)}$ ( R) for renaming both table R to S and its attributes from $A_1$, $A_2$, …, $A_n$ to $B_1$, $B_2$, … , $B_n$
- (ii) $\rho_s$( R) for renaming only table R to **S.**
- (iii) $\rho_{(B1, B2, \ldots, Bn)}$ ( R) for renaming only attributes of table R from $A_1$, $A_2$, …, $A_n$ to $B_1$, $B_2$, …, $B_n$
- Where rho ($\rho$ ) denotes RENAME operator, S is the new relation name and ($B_1$, $B_2$, …, $B_n$) are the new attribute names.
- E.g. if attributes of R are $A_1$, $A_2$, …, $A_n$, with $P_{S(B1, B2, \ldots, Bn)}$ ( R) , the relation is renamed S with new attributes $B_1$, $B_2$, …, $B_n$ for $A_1$, $A_2$, …, $A_n$
- Eg. For the rename of the result of the relational algebra operation, $\rho$ expression follows. R(Last_name, First_name, Salary) ← $\pi_{\text{LNAME, FNAME,SALARY}}$(EMPLOYEE)
- $\rho_{\text{R(Last\_name, First\_name, Salary)}}(\pi_{\text{LNAME, FNAME,SALARY}}(\text{EMPLOYEE}))$

# Unary Relational Operations: PROJECT

- The general form of the *project* operation is:

$$\pi_{\text{<attribute list>}}(R)$$

  - $\pi$ (pi) is the symbol used to represent the *project* operation
  - <attribute list> is the desired list of attributes from relation R.
- The project operation *removes any duplicate tuples*
  - This is because the result of the *project* operation must be a *set of tuples*
    - Mathematical sets *do not allow* duplicate elements.

# Unary Relational Operations: PROJECT

- PROJECT Operation Properties
  - The number of tuples in the result of projection $\pi_{<list>}(R)$ is always less or equal to the number of tuples in R
    - If the list of attributes includes a *key* of R, then the number of tuples in the result of PROJECT is *equal* to the number of tuples in R
  - PROJECT is *not* commutative
    - $\pi_{<list1>} (\pi_{<list2>} (R) ) = \pi_{<list1>} (R)$ as long as <list2> contains the attributes in <list1>

# Examples of applying SELECT and PROJECT operations

**Figure 8.1** Results of SELECT and PROJECT operations. (a) $\sigma_{(Dno=4 \text{ AND } Salary>25000) \text{ OR } (Dno=5 \text{ AND } Salary>30000)}$(EMPLOYEE). (b) $\pi_{Lname, Fname, Salary}$(EMPLOYEE). (c) $\pi_{Sex, Salary}$(EMPLOYEE).

(a)

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |

(b)

| Lname | Fname | Salary |
|---|---|---|
| Smith | John | 30000 |
| Wong | Franklin | 40000 |
| Zelaya | Alicia | 25000 |
| Wallace | Jennifer | 43000 |
| Narayan | Ramesh | 38000 |
| English | Joyce | 25000 |
| Jabbar | Ahmad | 25000 |
| Borg | James | 55000 |

(c)

| Sex | Salary |
|---|---|
| M | 30000 |
| M | 40000 |
| F | 25000 |
| F | 43000 |
| M | 38000 |
| M | 25000 |
| M | 55000 |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# 1.3 Relational Algebra Operations from Set Theory: UNION

- UNION Operation
  - Binary operation, denoted by $\cup$
  - The result of $R \cup S$, is a relation that includes all tuples that are either in R or in S or in both R and S
  - Duplicate tuples are eliminated
  - The two operand relations R and S must be "type compatible" (or UNION compatible)
    - R and S must have same number of attributes
    - Each pair of corresponding attributes must be type compatible (have same or compatible domains)

# 1.3 Relational Algebra Operations from Set Theory: UNION

- Example:
  - To retrieve the social security numbers of all employees who either *work in department 5* (RESULT1 below) or *directly supervise an employee who works in department 5* (RESULT2 below)
  - We can use the UNION operation as follows:

$$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO=5}} (\text{EMPLOYEE})$$

$$\text{RESULT1} \leftarrow \pi_{\text{SSN}}(\text{DEP5\_EMPS})$$

$$\text{RESULT2(SSN)} \leftarrow \pi_{\text{SUPERSSN}}(\text{DEP5\_EMPS})$$

$$\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$$

  - The union operation produces the tuples that are in either RESULT1 or RESULT2 or both

# **Figure 8.3** Result of the UNION operation RESULT ← RESULT1 ∪ RESULT2

**RESULT1**

| Ssn |
|---|
| 123456789 |
| 333445555 |
| 666884444 |
| 453453453 |

**RESULT2**

| Ssn |
|---|
| 333445555 |
| 888665555 |

**RESULT**

| Ssn |
|---|
| 123456789 |
| 333445555 |
| 666884444 |
| 453453453 |
| 888665555 |

# Relational Algebra Operations from Set Theory

- Type Compatibility of operands is required for the binary set operation UNION $\cup$, (also for INTERSECTION $\cap$, and SET DIFFERENCE –, see next slides)
- R1(A1, A2, ..., An) and R2(B1, B2, ..., Bn) are type compatible if:
  - they have the same number of attributes, and
  - the domains of corresponding attributes are type compatible (i.e. dom(Ai)=dom(Bi) for i=1, 2, ..., n).
- The resulting relation for R1$\cup$R2 (also for R1$\cap$R2, or R1–R2, see next slides) has the same attribute names as the *first* operand relation R1 (by convention)

# Relational Algebra Operations from Set Theory: INTERSECTION

- INTERSECTION is denoted by $\cap$
- The result of the operation $R \cap S$, is a relation that includes all tuples that are in both R and S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

# Relational Algebra Operations from Set Theory: SET DIFFERENCE

- SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by –
- The result of R – S, is a relation that includes all tuples that are in R but not in S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

# Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE

**Figure 8.4** The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT − INSTRUCTOR. (e) INSTRUCTOR − STUDENT.

(a) STUDENT

| Fn | Ln |
|---|---|
| Susan | Yao |
| Ramesh | Shah |
| Johnny | Kohler |
| Barbara | Jones |
| Amy | Ford |
| Jimmy | Wang |
| Ernest | Gilbert |

INSTRUCTOR

| Fname | Lname |
|---|---|
| John | Smith |
| Ricardo | Browne |
| Susan | Yao |
| Francis | Johnson |
| Ramesh | Shah |

(b)

| Fn | Ln |
|---|---|
| Susan | Yao |
| Ramesh | Shah |
| Johnny | Kohler |
| Barbara | Jones |
| Amy | Ford |
| Jimmy | Wang |
| Ernest | Gilbert |
| John | Smith |
| Ricardo | Browne |
| Francis | Johnson |

(c)

| Fn | Ln |
|---|---|
| Susan | Yao |
| Ramesh | Shah |

(d)

| Fn | Ln |
|---|---|
| Johnny | Kohler |
| Barbara | Jones |
| Amy | Ford |
| Jimmy | Wang |
| Ernest | Gilbert |

(e)

| Fname | Lname |
|---|---|
| John | Smith |
| Ricardo | Browne |
| Francis | Johnson |

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# Some properties of UNION, INTERSECT, and DIFFERENCE

- Notice that both union and intersection are *commutative* operations; that is
  - $R \cup S = S \cup R$, and $R \cap S = S \cap R$
- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative* operations; that is
  - $R \cup (S \cup T) = (R \cup S) \cup T$
  - $(R \cap S) \cap T = R \cap (S \cap T)$
- The minus operation is not commutative; that is, in general
  - $R - S \neq S - R$

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- CARTESIAN (or CROSS) PRODUCT Operation
  - This operation is used to combine tuples from two relations in a combinatorial fashion.
  - Denoted by R(A1, A2, . . ., An) x S(B1, B2, . . ., Bm)
  - Result is a relation Q with degree n + m attributes:
    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
  - The resulting relation state has one tuple for each combination of tuples—one from R and one from S.
  - Hence, if R has $n_R$ tuples (denoted as $|R| = n_R$ ), and S has $n_S$ tuples, then R x S will have $n_R * n_S$ tuples.
  - The two operands do NOT have to be "type compatible”

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- Generally, CROSS PRODUCT is not a meaningful operation
  - Can become meaningful when followed by other operations
- Example (not meaningful):
  - FEMALE_EMPS $\leftarrow \sigma_{SEX='F'}$(EMPLOYEE)
  - EMPNAMES $\leftarrow \pi_{\text{FNAME, LNAME, SSN}}$ (FEMALE_EMPS)
  - EMP_DEPENDENTS $\leftarrow$ EMPNAMES x DEPENDENT
- EMP_DEPENDENTS will contain every combination of EMPNAMES and DEPENDENT
  - whether or not they are actually related

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- To keep only combinations where the DEPENDENT is related to the EMPLOYEE, we add a SELECT operation as follows
- Example (meaningful):
  - FEMALE_EMPS $\leftarrow \sigma_{SEX='F'}$(EMPLOYEE)
  - EMPNAMES $\leftarrow \pi_{FNAME,\ LNAME,\ SSN}$ (FEMALE_EMPS)
  - EMP_DEPENDENTS $\leftarrow$ EMPNAMES x DEPENDENT
  - ACTUAL_DEPS $\leftarrow \sigma_{SSN=ESSN}$(EMP_DEPENDENTS)
  - RESULT $\leftarrow \pi_{FNAME,\ LNAME,\ DEPENDENT\_NAME}$ (ACTUAL_DEPS)
- RESULT will now contain the name of female employees and their dependents

# **Figure 8.5** The CARTESIAN PRODUCT (CROSS PRODUCT) operation.

**FEMALE_EMPS**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| Alicia | J | Zelaya | 999887777 | 1968-07-19 | 3321Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |

**EMPNAMES**

| Fname | Lname | Ssn |
|---|---|---|
| Alicia | Zelaya | 999887777 |
| Jennifer | Wallace | 987654321 |
| Joyce | English | 453453453 |

**EMP_DEPENDENTS**

| Fname | Lname | Ssn | Essn | Dependent_name | Sex | Bdate | ... |
|---|---|---|---|---|---|---|---|
| Alicia | Zelaya | 999887777 | 333445555 | Alice | F | 1986-04-05 | ... |
| Alicia | Zelaya | 999887777 | 333445555 | Theodore | M | 1983-10-25 | ... |
| Alicia | Zelaya | 999887777 | 333445555 | Joy | F | 1958-05-03 | ... |
| Alicia | Zelaya | 999887777 | 987654321 | Abner | M | 1942-02-28 | ... |
| Alicia | Zelaya | 999887777 | 123456789 | Michael | M | 1988-01-04 | ... |
| Alicia | Zelaya | 999887777 | 123456789 | Alice | F | 1988-12-30 | ... |
| Alicia | Zelaya | 999887777 | 123456789 | Elizabeth | F | 1967-05-05 | ... |
| Jennifer | Wallace | 987654321 | 333445555 | Alice | F | 1986-04-05 | ... |
| Jennifer | Wallace | 987654321 | 333445555 | Theodore | M | 1983-10-25 | ... |
| Jennifer | Wallace | 987654321 | 333445555 | Joy | F | 1958-05-03 | ... |
| Jennifer | Wallace | 987654321 | 987654321 | Abner | M | 1942-02-28 | ... |
| Jennifer | Wallace | 987654321 | 123456789 | Michael | M | 1988-01-04 | ... |
| Jennifer | Wallace | 987654321 | 123456789 | Alice | F | 1988-12-30 | ... |
| Jennifer | Wallace | 987654321 | 123456789 | Elizabeth | F | 1967-05-05 | ... |
| Joyce | English | 453453453 | 333445555 | Alice | F | 1986-04-05 | ... |
| Joyce | English | 453453453 | 333445555 | Theodore | M | 1983-10-25 | ... |
| Joyce | English | 453453453 | 333445555 | Joy | F | 1958-05-03 | ... |
| Joyce | English | 453453453 | 987654321 | Abner | M | 1942-02-28 | ... |
| Joyce | English | 453453453 | 123456789 | Michael | M | 1988-01-04 | ... |
| Joyce | English | 453453453 | 123456789 | Alice | F | 1988-12-30 | ... |
| Joyce | English | 453453453 | 123456789 | Elizabeth | F | 1967-05-05 | ... |

## **Figure 8.5 (continued)** The CARTESIAN PRODUCT (CROSS PRODUCT) operation.

**ACTUAL_DEPENDENTS**

| Fname | Lname | Ssn | Essn | Dependent_name | Sex | Bdate | . . . |
|---|---|---|---|---|---|---|---|
| Jennifer | Wallace | 987654321 | 987654321 | Abner | M | 1942-02-28 | . . . |

**RESULT**

| Fname | Lname | Dependent_name |
|---|---|---|
| Jennifer | Wallace | Abner |

# Binary Relational Operations: JOIN

- JOIN Operation (denoted by $\bowtie_{\text{<joincondition>}}$ )
  - The sequence of CARTESIAN PRODUCT followed by SELECT is used quite commonly to identify and select related tuples from two relations
  - A special operation, called JOIN combines this sequence into a single operation
  - This operation is very important for any relational database with more than a single relation, because it allows us *combine related tuples* from various relations
  - The general form of a join operation on two relations R(A1, A2, . . ., An) and S(B1, B2, . . ., Bm) is:

$$R \bowtie_{\text{<joincondition>}} S$$

  - where R and S can be any relations that result from general *relational algebra expressions*.

# Binary Relational Operations: JOIN (cont.)

- Example: Suppose that we want to retrieve the name of the manager of each department.
  - To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple.
  - We do this by using the join $\bowtie_{\text{<joincondition>}}$ operation.
  - DEPT_MGR $\leftarrow$ DEPARTMENT $\bowtie_{\text{MGRSSN=SSN}}$ EMPLOYEE
- MGRSSN=SSN is the join condition
  - Combines each department record with the employee who manages the department
  - The join condition can also be specified as DEPARTMENT.MGRSSN= EMPLOYEE.SSN

## **Figure 8.6** Result of the JOIN operation DEPT_MGR ← DEPARTMENT $\bowtie_{\text{Mgr\_ssn=Ssn}}$ EMPLOYEE

DEPT_MGR

| Dname | Dnumber | Mgr_ssn | ... | Fname | Minit | Lname | Ssn | ... |
|---|---|---|---|---|---|---|---|---|
| Research | 5 | 333445555 | ... | Franklin | T | Wong | 333445555 | ... |
| Administration | 4 | 987654321 | ... | Jennifer | S | Wallace | 987654321 | ... |
| Headquarters | 1 | 888665555 | ... | James | E | Borg | 888665555 | ... |

# Some properties of JOIN

- Consider the following JOIN operation:
  - $R(A1, A2, . . ., An) \bowtie_{R.Ai=S.Bj} S(B1, B2, . . ., Bm)$
  - Result is a relation Q with degree n + m attributes:
    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
  - The resulting relation state has one tuple for each combination of tuples—r from R and s from S, but *only if they satisfy the join condition* r[Ai]=s[Bj] or R.Ai = S.Bj
  - Hence, if R has $n_R$ tuples, and S has $n_S$ tuples, then the join result will generally have *less than* $n_R * n_S$ tuples.
  - Only related tuples (based on the join condition) will appear in the result

# Complete Set of Relational Operations

- The set of operations including SELECT $\sigma$, PROJECT $\pi$ , UNION $\cup$, DIFFERENCE $-$ , RENAME $\rho$, and CARTESIAN PRODUCT X is called a *complete set* because any other relational algebra expression can be expressed by a combination of these five operations.
- For example:
  - $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$
  - $R \bowtie_{\text{<join condition>}} S = \sigma_{\text{<join condition>}} (R \times S)$

# Examples of Queries in Relational Algebra – Single expressions

**Q1: Retrieve the name and address of all employees who work for the 'Research' department.**

$\pi_{\text{Fname, Lname, Address}} (\sigma_{\text{Dname= 'Research'}} (\text{DEPARTMENT} \bowtie_{\text{Dnumber=Dno}} (\text{EMPLOYEE})))$

# Division Operation (÷)

- DIVISION Operation
  - The DIVISION operation is useful for a special kind of query as: Retrieve the names of employees who work on all the projects that ‘John Smith’ works on. That is, if John Smith works on the set of projects with Pno = {1, 2}, any employee selected must have worked on all the Pnos in this set.
  - The division operation is applied to two relations
  - R(Z) ÷ S(X) are the two input relation operands of the division operator and the resulting relation is T(Y). For example, Works_on(Essn,Pno) ÷ D(Pno) will give result containing all Essns who have worked on all Pno’s in D(Pno) set.
  - For R(Z) ÷ S(X), denominator relation has its set of attributes X (eg. S(a: varchar2(2))) as a subset of the numerator relation’s set of attributes, Z (a: varchar2(2), b: varchar2(2)). The resulting relation, T(Y) has the set of attributes Y = Z - X (eg. b: varchar2(2)) which is the set of attributes of R that are not attributes of S.
  - The result of this DIVISION is a relation T(Y) that includes a tuple t that must appear in the result T if tuples $t_R$ appear in the numerator relation R in combination with every tuple in the denominator relation S.

# Division Operation (÷) and (natural join operator *)

- We can answer this query as:
- (a) Get Denomenator (Pnos worked by John Smith) as:
  Smith_Pnos ← $\pi_{\text{Pno}}$ ($\sigma_{\text{Lname='Smith' and Fname= 'John'}}$ ( WORKS_ON $\bowtie_{\text{Essn=Ssn}}$ EMPLOYEE))
  (b) Get Numerator (Essn with Pnos worked by all employees) as:
  Ssn_Pnos ← $\pi_{\text{Essn, Pno}}$ (WORKS_ON)
  (c) Get all employees who worked on all project worked on by Smith as:
  SSNs(Ssn) ← Ssn_Pnos ÷ Smith_Pnos
- See the result of these operations in Fig. 8.8 on page 256 of book.
- Note that the natural join (*) which is a join of two tables on join foreign/primary key attributes (e.g., Ssn) with the same name can be used for example to get the names of these employees working on all projects worked on by Smith as: $\pi_{\text{Fname, Lname}}$(SSNs * EMPLOYEE)

# Fig 8.8: Division Operation (÷)

**Figure 8.8** The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) T ← R ÷ S.

(a)

SSN_PNOS

| Essn | Pno |
|---|---|
| 123456789 | 1 |
| 123456789 | 2 |
| 666884444 | 3 |
| 453453453 | 1 |
| 453453453 | 2 |
| 333445555 | 2 |
| 333445555 | 3 |
| 333445555 | 10 |
| 333445555 | 20 |
| 999887777 | 30 |
| 999887777 | 10 |
| 987987987 | 10 |
| 987987987 | 30 |
| 987654321 | 30 |
| 987654321 | 20 |
| 888665555 | 20 |

SMITH_PNOS

| Pno |
|---|
| 1 |
| 2 |

SSNS

| Ssn |
|---|
| 123456789 |
| 453453453 |

(b)

R

| A | B |
|---|---|
| a1 | b1 |
| a2 | b1 |
| a3 | b1 |
| a4 | b1 |
| a1 | b2 |
| a3 | b2 |
| a2 | b3 |
| a3 | b3 |
| a4 | b3 |
| a1 | b4 |
| a2 | b4 |
| a3 | b4 |

S

| A |
|---|
| a1 |
| a2 |
| a3 |

T

| B |
|---|
| b1 |
| b4 |


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# Additional Relational Operations: Aggregate Functions and Grouping

- A type of request that cannot be expressed in the basic relational algebra is to specify mathematical **aggregate functions** on collections of values from the database.
- Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples.
  - These functions are used in simple statistical queries that summarize information from the database tuples.
- Common functions applied to collections of numeric values include
  - SUM, AVERAGE, MAXIMUM, MINIMUM and COUNT.
- The COUNT function is used for counting tuples or values.

# Aggregate Function Operation

- Use of the Aggregate Function operation $\Im$ (called script)
- The general format that includes grouping attributes is:
- <grouping attributes> $\Im_{\text{<function list>}}$ (R)
  where <grouping attributes> is a list of attributes of relation R, <function list> is a list of (<function><attribute>) pairs. Function is one of SUM, AVERAGE, MAXIMUM, MINIMUM, COUNT. The result has the grouping attributes plus one attribute for each element in the function list.
- $\Im_{\text{MAX Salary}}$ (EMPLOYEE) retrieves the maximum salary value from the EMPLOYEE relation
  - $\Im_{\text{MIN Salary}}$ (EMPLOYEE) retrieves the minimum Salary value from the EMPLOYEE relation
  - $\Im_{\text{SUM Salary}}$ (EMPLOYEE) retrieves the sum of the Salary from the EMPLOYEE relation
  - $\Im_{\text{COUNT SSN, AVERAGE Salary}}$ (EMPLOYEE) computes the count (number) of employees and their average salary
    - Note: count just counts the number of rows, without removing duplicates

# Using Grouping with Aggregation

- The previous examples all summarized one or more attributes for a set of tuples
  - Maximum Salary or Count (number of) Ssn
- Grouping can be combined with Aggregate Functions
- Example: For each department, retrieve the DNO, COUNT SSN, and AVERAGE SALARY
- A variation of aggregate operation $\Im$ allows this:
  - Grouping attribute placed to left of symbol
  - Aggregate functions to right of symbol
  - $_{\text{DNO}}\ \Im\ _{\text{COUNT SSN, AVERAGE Salary}}$ (EMPLOYEE)
- Above operation groups employees by DNO (department number) and computes the count of employees and average salary per department

# Figure 8.10 The aggregate function operation.

a. $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(_{\text{Dno}}\ \Im\ _{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE}))$.

b. $_{\text{Dno}}\ \Im\ _{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

c. $\Im\ _{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

- a is renamed with ρ, b has no renaming and c has no grouping.

(a) R

| Dno | No_of_employees | Average_sal |
|---|---|---|
| 5 | 4 | 33250 |
| 4 | 3 | 31000 |
| 1 | 1 | 55000 |

(b)

| Dno | Count_ssn | Average_salary |
|---|---|---|
| 5 | 4 | 33250 |
| 4 | 3 | 31000 |
| 1 | 1 | 55000 |

(c)

| Count_ssn | Average_salary |
|---|---|
| 8 | 35125 |

# 2. Relational Calculus

- Relational Calculus is another formal query language for the relational model.
- Two variations of it are:
  - tuple relational calculus and
  - domain relational calculus.
- In both variations one declarative expression is written to specify a retrieval query.
- The expression has no description of how, or in what order, to evaluate a query.
- A calculus expression specifies what is to be retrieved rather than how to retrieve it.
- Relational calculus is a nonprocedural language as opposed to the relational algebra that is procedural.
- A calculus expression may be written in different ways that do not determine how the query is evaluated.

# 2. Relational Calculus

- Any retrieval that can be specified in the relational algebra can also be specified in relational calculus, and vice versa.
- A relational query language L is relationally complete if we can express in L any query that can be expressed in relational calculus.
- This relational completeness property is used as a basis for comparing the expressive power of high-level query languages.
- Most languages such as SQL are relationally complete but have more expressive power than relational algebra or relational calculus:
    - as they have additional operations like aggregate functions, grouping and ordering.

# Tuple Relational Calculus

- The tuple relational calculus (TRC) is based on specifying a number of tuple variables.
- Each tuple variable usually ranges over a particular database relation, meaning that the variable may take as its value any individual tuple from that relation.
- A simple tuple relational calculus query is of the form **{t | COND(t)}**
- **Expressed in general also as: $\{t_1.A_j, t_2.A_k, \ldots, t_n.A_m \mid COND(t_1, t_2, \ldots, t_{n+m})\}$**
- where t is a tuple variable and COND (t) is a conditional expression involving t. Also, the $\mathbf{t_i}$'s are tuple variables and $\mathbf{A_j}$'s are attributes of the relation on which $\mathbf{t_i}$ ranges.
- COND is a condition (atom or formula) of TRC of the form:
- (i) Relation ($\mathbf{t_i}$), eg. Employee(t)
- (ii) **$t_1$.A op $t_1$.B, eg. E.ssn = d.Essn**
- **(iii) $t_1$.A op c or c op $t_1$.A, eg. Salary > 30000**
- **(iv) A formula (F) is made up of atoms and atoms are connected with logical ops and quantifiers ($\exists$, $\forall$), eg. $F_1$ AND $F_2$, $F_1$ OR $F_2$ , NOT($F_1$,** ($\exists$ t)(F) and ($\forall$ t)(F).
- The result of such a query is the set of all tuples t that satisfy COND (t).

# Tuple Relational Calculus

- Example: To find the first and last names of all employees whose salary is above $50,000, we can write the following tuple calculus expression:

**{t.FNAME, t.LNAME | EMPLOYEE(t) AND t.SALARY>50000}**

- The condition EMPLOYEE(t) specifies that the **range relation** of tuple variable t is EMPLOYEE.
- The first and last name (PROJECTION in relational algebra ($\pi_{FNAME,\ LNAME}$)) of each EMPLOYEE tuple t that satisfies the condition t.SALARY>50000 (SELECTION in relational algebra ($\sigma_{SALARY\ >50000}$)) will be retrieved.

# The Existential and Universal Quantifiers

- Two special symbols called quantifiers can appear in formulas; these are the universal quantifier ($\forall$) and the existential quantifier ($\exists$).
- Informally, a tuple variable t is bound if it is quantified, meaning that it appears in an ($\forall$ t) or ($\exists$ t) clause; otherwise, it is free.
- If F is a formula, then so are ($\exists$ t)(F) and ($\forall$ t)(F), where t is a tuple variable.
  - The formula ($\exists$ t)(F) is true if the formula F evaluates to true for some (at least one) tuple assigned to free occurrences of t in F; otherwise ($\exists$ t)(F) is false.
  - The formula ($\forall$ t)(F) is true if the formula F evaluates to true for every tuple (in the universe) assigned to free occurrences of t in F; otherwise ($\forall$ t)(F) is false.

# Example Query Using Existential Quantifier

- Retrieve the name and address of all employees who work for the 'Research' department. The query can be expressed as :

**{t.FNAME, t.LNAME, t.ADDRESS | EMPLOYEE(t) and (∃ d) (DEPARTMENT(d) and d.DNAME='Research' and d.DNUMBER=t.DNO) }**

- The only *free tuple variables* in a relational calculus expression should be those that appear to the left of the bar ( | ).
  - In above query, t is the only free variable; it is then *bound successively* to each tuple.
- If a tuple *satisfies the conditions* specified in the query, the attributes FNAME, LNAME, and ADDRESS are retrieved for each such tuple.
  - The conditions EMPLOYEE (t) and DEPARTMENT(d) specify the range relations for t and d.
  - The condition d.DNAME = 'Research' is a selection condition and corresponds to a SELECT operation in the relational algebra, whereas the condition d.DNUMBER = t.DNO is a JOIN condition.